Approximute Quadrature of the Circle. By Pliny Earle Chase, LL.D. (Read before the American Philosophical Society, June 20th, 1879.)


$$
\mathrm{AB}=3 ; \Lambda \mathrm{C}=20
$$

$$
\mathrm{AD}=3 \mathrm{AB} ; \Lambda \mathrm{X}=3 \mathrm{AC}
$$

BE parallel to CD

$$
\mathbf{E Y}=\mathbf{A C}
$$

$$
\mathrm{XY}=3.141585 \mathrm{AC}
$$

The deviation from perfect accuracy is less than $\frac{x^{2} \delta \sigma}{}$ of one per cent. which would give un error of less than $\frac{1}{6}$ of an inch per mile. For all practical purposes the construction may be regarded as exact, for the error would be inappreciable in any mechanical work.

Haverford College, June 16th, $18 \% 9$.
Note.-Jtily 16, 1879. My attention has been called to the following more-complicated construction, and closer approximation, in Perkins's Geometry (D. Appleton \& Co., 1853).

On an indefinite straight line $\mathbf{A} \mathbf{N}$, take $\mathbf{A B}=\mathrm{BD}=\mathrm{DE}=1$; at E erect a perpendicular $\mathrm{EG}=2 \mathrm{~A} B=2 \mathrm{EF}$; on EN take $\mathrm{EH}=\mathrm{HK}$ $=\mathrm{A} G, \mathrm{KL}($ towards $\Lambda)=\mathrm{AF}, \mathrm{L} M($ towards N$)=\mathrm{DG}, \mathrm{MN}=\mathrm{DF}$; bisect L N at $\mathrm{P}, \mathrm{E} \mathrm{P}$ at $\mathrm{R}, \mathrm{A} \mathrm{B}$ at C ; trisect ER at T . Then $\mathrm{C} \mathrm{T}=$ 3.1415922 .

The author calls this method "very simple," and says, that a better one "cim hardly be expected, or even desired." But the approximation of Adrian Metius, $\frac{355}{113}$, is still closer, and the following construction of his ratio is simpler.


On A B=7 erect the perpendicular $\mathrm{B} C=8$; extend CB to D , making $\mathrm{B} D=9$; on A D erect the perpendicular $\mathrm{DF}=15$; take $\mathrm{A} \mathrm{E}=\mathrm{A} \mathrm{C}$, and draw E G parallel to F C. Then $\frac{\mathrm{AF}}{\mathrm{AG}}=\frac{355}{113}=3.1415929$, the true ratio being $3.1415926+$.

The error of this construction is less than II $\frac{1}{2} \sigma 00$ of one per cent. Perkins's error is more than robod of one per cent. Neither method is so simple, nor so desirable for practical purposes, as the one which I communicated to the Society at its June meeting.

